2013 IREP Symposium-Bulk Power System Dynamics and Control –IX (IREP), August 25-30, 2013, Rethymnon, Greece

An Application of High Performance Computing to Transmission Switching

Anthony Papavasiliou — CORE, UCL

Shmuel S. Oren, Zhu Yang — IEOR, UC Berkeley

Pranav Balasubramanian, Kory Hedman — ECE, ASU

Abstract

We present a parallel implementation of three transmission switching algorithms. The first is based on a parallel search of all candidate lines, the second is based on a priority listing of lines and the third is based on decomposing the set of candidate lines in smaller subsets that are solved in parallel. We present a duality result that justifies the priority listing of the second algorithm. The We apply the algorithms on an IEEE 118 bus system and compare the relative performance of the algorithms in terms of cost of system operations. We envision the use of high performance computing for implementing the proposed algorithms in a real-time setting in order to exploit topology control of transmission networks for operating the system at minimum cost and for responding to system contingencies.

I. Introduction

Topology control, or transmission switching, is the problem of optimizing the transmission network configuration of an electric power system in order to achieve a certain objective such as the minimization of load losses in an emergency event or the minimization of operating costs. There are numerous examples of transmission switching in practice in response to emergency events, however these actions take place currently on an ad hoc basis. The first formal treatment of the problem as a large-scale optimization problem was introduced by Fisher [1]. In its full scale, the problem is extremely challenging computationally. The authors in [2] address the problem with Benders decomposition. The authors in [3] propose heuristics for solving the transmission switching problem based on ranking lines on the basis of the LMP difference between the nodes that the lines are adjacent to. The proposed algorithms are demonstrated to be effective in the IEEE 118 bus model.

Topology control can be perceived as the transmission network analog of unit commitment. The ability to actively control the transmission network introduces additional degrees of freedom in the control space of the system operator that can enhance economic efficiency and reliability. It may seem counterintuitive that switching lines can improve the cost of power system operations. The source of such efficiency stems from the fact that transmission networks have been designed with an emphasis on redundancy, in order to ensure reliability. Under certain loading conditions, certain lines may increase the cost of operations although under different loading conditions these same lines may be necessary in order to ensure that demand is satisfied. In the future, the ability to actively switch transmission lines can augment the existing ability of the operator to control the network by committing and shutting down generators.

Unit commitment is performed in a systematic fashion in the day-ahead, intra-day and real-time scheduling process through the solution of the unit commitment and economic dispatch model. Existing computational technology is capable of delivering near-optimal unit commitment schedules within an acceptable time frame. In contrast, topology control is currently performed on an ad-hoc basis, based on operator experience. The development of algorithms that are capable of delivering efficient transmission switching control actions within an operationally acceptable time frame can support the adoption of systematic topology control in power system operations. Parallel computing can support the development of scalable topology control algorithms.

Parallel computing has a rich history in the area of power systems planning and operations. A review of the application of high performance computing in power systems is presented by Falcao [4]. Pereira et al. [5] present the application of distributed computing in reliability evaluation for composite outages, scenario analysis for hydro dominated systems and security-constrained dispatch. Monticelli et al. [6] formulate the security-constrained optimal power flow with corrective rescheduling and demonstrate economic benefits in the dispatch on an IEEE system with 118 buses. Kim and Baldick [7] present a parallel algorithm for solving distributed optimal power flow. Bakirtzis and Biskas [8] propose a decentralized Lagrangian relaxation algorithm

978-1-4799-0199-9/13/$31.00 ©2013 IEEE

for solving the optimal power flow problem presented by Kim and Baldick [7]. A parallel implementation of the algorithm in Bakirtzis and Biskas [8] using PVM is presented by Biskas et al. [9].

In this paper we implement three heuristic algorithms in parallel. One can envision that with the advent of distributed computing, such computations could assist in day-ahead as well as emergency situations within operational time frames.

II. Model Description

A. A Mixed Integer Linear Programming Formulation of the DCOPF

We present a single-period economic dispatch model with an objective of minimizing operating costs. Binary decision variables are used for modeling topology control, and continuous production decision variables are used for deciding the power output of each unit. Kirchoff's voltage and current laws are linearized and we ignore losses and reactive power flows. The resulting direct current optimal power flow with transmission topology control can be described as a mixed integer linear program. In the following model, Eqs. (2), (3) impose maximum and minimum capacity limits on generating units. Eq. (4) is the power balance equation for each bus. Eqs. (5), (6) impose transmission constraints on the problem. Eqs. (7), (8) are Kirchhoff's linearzied DC power flow equations. The binary decision variable z_k indicates whether a line is on ($z_k = 1$) or off ($z_k = 0$). The big-M parameter in Eqs. (7), (8) ensures that when a line is off, Kirchoff's laws are not active and Eqs. (5), (6) ensure that the flow in the line is equal to zero. Note that load shedding can be included in this formulation by introducing an auxiliary `load shedding' unit with a marginal cost equal to the value of lost load, a minimum capacity of 0 and a maximum capacity equal to the demand in a given bus. The decision variables and parameters of the model are described in detail in the appendix.

$$(TXIP): \min \sum_{g \in G} C_g p_g \quad (1)$$
$$p_g - P_g^+ \leq 0, g \in G \quad (2)$$
$$-p_g + P_g^- \leq 0, g \in G \quad (3)$$
$$-\sum_{k \in K:F(k)=n} f_k + \sum_{k \in K:T(k)=n} f_k + \sum_{g \in G_n} p_g - D_n = 0, n \in N \quad (4)$$
$$-f_k - z_k TC_k \leq 0, k \in K \quad (5)$$
$$f_k - z_k TC_k \leq 0, k \in K \quad (6)$$
$$f_k - B_k(\theta_m - \theta_n) - M_k(1 - z_k) \leq 0, k = (m,n) \in K \quad (7)$$
$$-f_k + B_k(\theta_m - \theta_n) - M_k(1 - z_k) \leq 0, k = (m,n) \in K \quad (8)$$
$$p_g \geq 0, g \in G, z_k \in \{0,1\}, k \in K$$

We formulate the model using susceptances, B_k, instead of power transfer distribution factors (PTDFs). The drawback of using PTDFs in this model is that PTDFs change whenever the transmission system topology changes. Instead, line susceptances stem from the electric characteristics of lines and do not depend on control actions, thereby rendering them appropriate for topology control models as well as models where transmission outages are represented explicitly [10].

B. A Reformulation of the DCOPF with Fixed Switching Decisions

We present the following formulation of the DCOPF which will prove useful in the following exposition. In the following models we also define dual variables.

$$(TXLP): \min \sum_{g \in G} C_g p_g \quad (9)$$
$$p_g - P_g^+ \leq 0, g \in G, (\mu_g^+) \quad (10)$$
$$-p_g + P_g^- \leq 0, g \in G, (\mu_g^-) \quad (11)$$
$$-\sum_{k \in K:F(k)=n} f_k + \sum_{k \in K:T(k)=n} f_k + \sum_{g \in G_n} p_g - D_n = 0, n \in N, (\rho_n) \quad (12)$$
$$-f_k - Z_k TC_k \leq 0, k \in K, (\lambda_k^-) \quad (13)$$
$$f_k - Z_k TC_k \leq 0, k \in K, (\lambda_k^+) \quad (14)$$
$$f_k - Z_k B_k(\theta_m - \theta_n), k = (m,n) \in K, (\psi_k) \quad (15)$$
$$p_g \geq 0, g \in G$$

We partition the set of lines $K = \bar{K} \cup \hat{K}$ in the network between the set of lines, $\bar{K}$, that are out of service at the moment the model is solved and the set of lines, $\hat{K}$, that are in service at the moment the model is solved. In contrast to the mixed integer linear program of the previous section, this problem represents a linear program where we have decided in advance on the switching status of lines.

Following the idea of Fuller [3], in order to come up with a reasonable criterion for selecting which lines to switch, we would like to know the sensitivity of the optimal cost on the switching action. For this reason, we express the linear program (TXLP) equivalently as the following nonlinear program:

$$(TXNLP): \min \sum_{g \in G} C_g p_g \quad (16)$$
$$p_g - P_g^+ \leq 0, g \in G, (\mu_g^+) \quad (17)$$
$$-p_g + P_g^- \leq 0, g \in G, (\mu_g^-) \quad (18)$$
$$-\sum_{k \in K:F(k)=n} f_k + \sum_{k \in K:T(k)=n} f_k + \sum_{g \in G_n} p_g - D_n = 0, n \in N, (\rho_n) \quad (19)$$
$$-f_k - s_k TC_k \leq 0, k \in \bar{K}, (\lambda_k^-) \quad (20)$$
$$f_k - s_k TC_k \leq 0, k \in \bar{K}, (\lambda_k^+) \quad (21)$$
$$-f_k - (1 - s_k) TC_k \leq 0, k \in \hat{K}, (\lambda_k^-) \quad (22)$$
$$f_k - (1 - s_k) TC_k \leq 0, k \in \hat{K}, (\lambda_k^+) \quad (23)$$
$$f_k - s_k B_k(\theta_m - \theta_n), k = (m,n) \in \bar{K}, (\psi_k) \quad (24)$$
$$f_k - (1 - s_k) B_k(\theta_m - \theta_n), k = (m,n) \in \hat{K}, (\psi_k) \quad (25)$$
$$s_k = 0, k \in K, (\gamma_k) \quad (26)$$
$$p_g \geq 0, g \in G$$

The control variable s_k represents a line switching action. The control action $s_k = 1$ switches the state of line k, whereas $s_k = 0$ keeps the line in its existing state. Thus, $s_k = 1$ for $k \in \overline{K}$ implies that line k is switched on, $s_k = 0$ for $k \in \overline{K}$ implies that line k is kept off, $s_k = 1$ for $k \in \widehat{K}$ implies that line k is switched off and $s_k = 0$ for $k \in \widehat{K}$ implies that line k is kept on. In this model we introduce the constraints of Eqs. (26) that fix the switching decisions s_k to 0, which implies that (TXLP) and (TXNLP) have the same primal optimal solution. This also implies that strict duality is satisfied for (TXNLP), and there is therefore a cost sensitivity interpretation to the dual multipliers of (TXNLP) at the primal optimal point.

The following identity can be established using the KKT conditions of this model. This result generalizes a result by Fuller [3] for the case where lines can also be switched out of service.

$$\gamma_k = TC_k((\lambda_k^+)^* + (\lambda_k^-)^*) + B_k(\theta_m^* - \theta_n^*)\psi_k^*, k \in \overline{K} \quad (27)$$
$$\gamma_k = f_k^*(\rho_n^* - \rho_m^*)\psi_k^*, k \in \widehat{K} \quad (28)$$

where the starred variables are the optimal primal and dual variables of (TXLP). Note, then, that the sensitivity of the switching decision, γ_k, which is defined in (TXNLP), does not require solving (TXLP) but instead can be derived from the primal-dual optimal pair of (TXLP). This can accelerate the computation of candidate lines for switching significantly, bringing computation times down to an operationally acceptable time frame. Note that the most promising candidate for switching according to this sensitivity criterion is the line with the lowest (most negative) γ_k and vice versa.

III. Solution Approach

In this section we present three parallel algorithms for heuristic topology control.

A. Greedy Line Selection

The first algorithm is a direct enumeration of all the lines. We select the line switching action which results in the greatest improvement. We iterate until we can find no improving switching action. The algorithm is shown in Fig. 1.

B. Greedy Line Selection with Priority Listing

The second algorithm is a ranking of the lines according to the sensitivity computed in Eqs. (27), (28). In this algorithm we implement the first switching action that results in an improvement. The algorithm schematic is shown in Fig. 2.

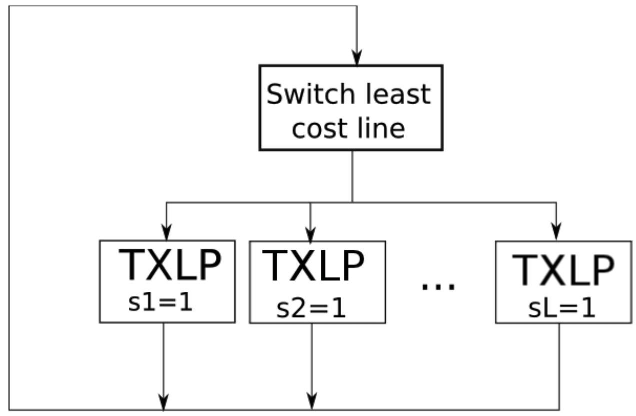

Fig. 1: Greedy line selection.

The derivation of the duality criterion of Eqs. (27), (28) stems from the KKT conditions of (TXLP):

$$0 \le \mu_g^+ \perp -p_g + P_g^+ \ge 0 \quad (29)$$
$$0 \le \mu_g^- \perp p_g - P_g^- \ge 0 \quad (30)$$
$$-\sum_{k\in K:F(k)=n} f_k + \sum_{k\in K:T(k)=n} f_k + \sum_{g\in G_n} p_g - D_n = 0, n \in N \quad (31)$$
$$0 \le \lambda_k^- \perp f_k + Z_k TC_k \ge 0 \quad (32)$$
$$0 \le \lambda_k^+ \perp -f_k + Z_k TC_k \ge 0 \quad (33)$$
$$f_k - Z_k B_k(\theta_m - \theta_n) = 0 \quad (34)$$
$$0 \le p_g \perp C_g + \mu_g^+ - \mu_g^- + \rho_n \ge 0 \quad (35)$$
$$\psi_k - \lambda_k^- + \lambda_k^+ - \rho_m + \rho_n = 0 \quad (36)$$
$$\sum_{k\epsilon K:T(k)=n} B_k Z_k \psi_k - \sum_{k\epsilon K:F(k)=n} B_k Z_k \psi_k = 0 \quad (37)$$

and the corresponding KKT conditions for (TXNLP):

$$0 \le \mu_g^+ \perp -p_g + P_g^+ \ge 0 \quad (38)$$
$$0 \le \mu_g^- \perp p_g - P_g^- \ge 0 \quad (39)$$
$$-\sum_{k\in K:F(k)=n} f_k + \sum_{k\in K:T(k)=n} f_k + \sum_{g\in G_n} p_g - D_n = 0, n \in N \quad (40)$$
$$0 \le \lambda_k^- \perp f_k + s_k TC_k \ge 0, k\epsilon\overline{K} \quad (41)$$
$$0 \le \lambda_k^- \perp f_k + (1 - s_k)TC_k \ge 0, k\epsilon\widehat{K} \quad (42)$$
$$0 \le \lambda_k^+ \perp -f_k + s_k TC_k \ge 0, k\epsilon\overline{K} \quad (43)$$
$$0 \le \lambda_k^+ \perp -f_k + (1 - s_k)TC_k \ge 0, k\epsilon\widehat{K} \quad (44)$$
$$f_k - s_k B_k(\theta_m - \theta_n) = 0, k\epsilon\overline{K} \quad (45)$$
$$f_k - (1 - s_k)B_k(\theta_m - \theta_n) = 0, k\epsilon\widehat{K}$$
$$s_k = 0 \quad (47)$$
$$0 \le p_g \perp C_g + \mu_g^+ - \mu_g^- + \rho_n \ge 0 \quad (48)$$
$$\psi_k - \lambda_k^- + \lambda_k^+ - \rho_m + \rho_n = 0 \quad (49)$$
$$\sum_{k\epsilon\overline{K}:T(k)=n} B_k s_k \psi_k + \sum_{k\epsilon\widehat{K}:T(k)=n} B_k(1 - s_k)\psi_k - \sum_{k\epsilon\overline{K}:F(k)=n} B_k s_k \psi_k - \sum_{k\epsilon\widehat{K}:T(k)=n} B_k(1 - s_k)\psi_k = 0 \quad (50)$$
$$-TC_k(\lambda_k^+ + \lambda_k^-) - B_k(\theta_m - \theta_n)\psi_k + \gamma_k = 0, k\epsilon\overline{K} \quad (51)$$
$$TC_k(\lambda_k^+ + \lambda_k^-) + B_k(\theta_m - \theta_n)\psi_k + \gamma_k = 0, k\epsilon\widehat{K} \quad (52)$$

Note that, because of the constraint $s_k = 0$ in (TXNLP), the KKT conditions of (TXLP) at the optimal solution are

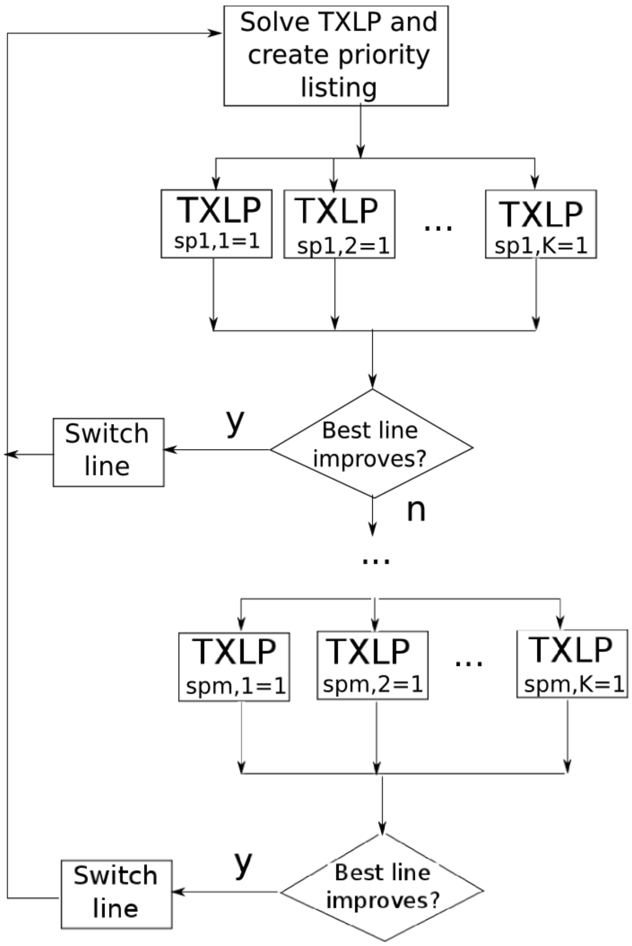

Fig. 2: Greedy line selection with priority listing.

identical with those of (TXNLP), with the additional equality constraint defining γ_k, Eqs. (51), (52). This allows us to use the primal and dual optimal solution of (TXLP) for computing the switching sensitivity of (TXNLP). The derivation of Eqs. (27), (28) follows from the KKT conditions.

C. MIP Heuristic

The MIP heuristic algorithm partitions the set of lines to mutually exclusive groups of lines that are candidate for switching. Smaller instances of (TXIP) are then solved. These smaller instances of (TXIP) are solved in parallel, and at each iteration of the algorithm the least cost solution is stored as the starting point for the next iteration. The schematic of the algorithm is shown in Fig. 3. The algorithm interrupts when no improvement can be achieved.

IV. Results

We have tested the three parallel algorithms presented in Section III on the IEEE 118 bus system as well as an

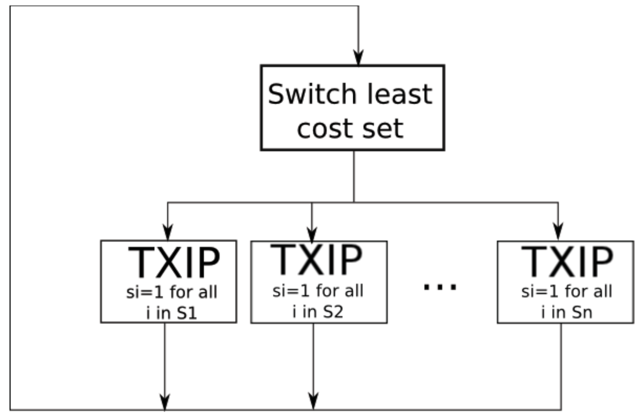

Fig. 3. MIP heuristic.

industrial scale model of the Pennsylvania Jersey Maryland (PJM) system made available by the Federal Energy Regulatory Commission (FERC). All algorithms were implemented in the Java callable library of CPLEX 12.4, and parallelized using the Message Passing Interface (MPI).

A. The IEEE 118 Bus System

The IEEE system consists of 118 buses, 19 generators, and 186 lines. We present the sequence of actions from iteration to iteration, as well as the cost improvement over iterations.

For (TX2) we have 2 processors at each iteration, which implies that two candidate switches are checked in parallel at a time, until either an improvement is detected or the full set of candidate lines is checked. For (TX3), we have partitioned the entire set of lines in the system to 9 groups of 18 lines (L1 - L18, L19 - L36 and so on) and one group of 24 lines (L163 - L186).

We note that in terms of performance (TX3) outperforms (TX2) which outperforms (TX1). It may be unintuitive that (TX2) should outperform (TX1), since (TX1) checks all lines at each iteration. L132, L136, L153 and L162 are switched by both (TX1) and (TX3). L132 and L153 are switched by both (TX2) and (TX3). L132 is switched by both (TX1) and (TX2). Lines L132 and L153 are switched by all algorithms.

The elapsed time for (TX1) was 1,314 seconds. The elapsed time for (TX2) was 300 seconds. The elapsed time for (TX3) was 17 seconds. All algorithms were run in parallel between 3 processors.

B. The FERC 13,867 Bus System

We obtained a data set from Federal Energy and Regulatory Commission, which is representative of, but not an exact replica of, the PJM Regional Transmission Organization. The system consists of 13,867 buses, 1,011 generators and 18,824 branches. Compared to the IEEE test case, the system is a much more realistic representation of an industrial scale network. The algorithms have been implemented on a high performance computing cluster in the Lawrence Livermore National Laboratory, with 8 CPUs per node, 2.4 GHz and 10 GB per node.

It.	TX1	Cost	TX2	Cost	TX3	Cost
1	L153	1924.5	L132	1930.6	L129 L132 L136	1778.3
2	L132	1795.9	L163	1797.5	L148 L153 L161 L162	1549.0
3	L136	1629.8	L133	1714.7		
4	L162	1607.9	L153	1683.0		
5	L37	1603.1	L151	1609.4		
6	L122	1600.3	L78	1600.6		
7	L14	1597.0	L85	1596.6		
8	L31	1595.9	L82	1596.1		
9	L19	1595.8	L96	1595.3		
10	L54	1595.6	L45	1595.32		
11	L60	1595.6	L48	1595.3		
12	L68	1595.6	L59	1595.3		

Table 1: Switching actions and costs by each of the transmission switching algorithms.

Due to its scale, the FERC model present significant computational challenges. Running (TXLP) itself takes up to an hour using CPLEX default settings, while the solution of (TXIP) is much greater even in the case where only a subset of lines are considered for switching. As a first step, we only implemented (TX1) and (TX2) with this system. The results of (TX1) and (TX2) are presented in Table 2, including the cost reduction and line switch sequence.

Due to running time constraints, we only performed 10 iterations. We note that with 10 lines open, both algorithms results in less than 2% cost reduction. We also note that there is no common line being switched in the two algorithms. Even though (TX1) outperforms (TX2), the running time of (TX2) is much less than the running time of (TX1) since (TX2) only checks a subset of all lines at each iteration. In addition, preliminary results show that reducing the number of lines checked before implementing the line switch in (TX2) can effectively reduce the solution time without affecting performance substantially. The system operator can choose an appropriate value of this parameter to balance performance and computation time. The preliminary results presented in this paper suggest various directions of future improvement. One promising direction of future research is warm starting the linear programming solver when solving a sequence of (TXLP).

It.	TX1	Cost impr. (%)	TX2	Cost impr. (%)
1	L17230	0.182	L2813	0.098
2	L2913	0.353	L1831	0.200
3	L8731	0.792	L11231	0.226
4	L12031	0.991	L103	0.441
5	L7031	1.404	L7482	0.605
6	L721	1.420	L2310	0.893
7	L293	1.556	L14823	1.030
8	L7981	1.652	L5567	1.059
9	L10002	1.762	L787	1.255
10	L8310	1.860	L8313	1.268

Table 2: Switching actions and cost reduction by (TX1) and (TX2) (FERC Data Set).

V. Conclusions

In this paper we have presented three parallel implementations for the optimal topology control in transmission networks. The first algorithm is a greedy method that sequentially switches the line with the greatest impact in reducing cost. Parallelization can take place in the search of the most effective line. The second algorithm ranks line first based on the sensitivity of the switching action, and switches a line once an improvement is detracted. Parallelization can again occur during the search of a line that improves cost performance. The third algorithm partitions the line set to subsets of lines that can be switched, and solves smaller instances of the transmission switching problem, selecting at each iteration the most efficient group. The algorithm is parallelized in the simultaneous solution of the transmission switching problems.

The algorithms have been tested and compared on the IEEE 118 bus system. The third algorithms is found to outperform the second algorithm which is found to outperform the first algorithm both in terms of cost as well as in terms of running time. We also present preliminary results obtained by implementing the algorithms in the Lawrence Livermore National Laboratory High Performance Computing Center, applied on an industrial scale model of the PJM system with 13,867 buses. These preliminary results suggest that further research is required in order to obtain solution times that are operationally acceptable for industrial scale systems.

Appendix

A. Notation

Sets

G: set of all generators

G_n: subset of generators connected in bus n

N: set of buses

K: set of lines

$\bar{K}$: set of lines that are out of service

$\hat{K}$: set of lines that are in service

Decision variables

s_k: decision to switch status of line k

z_k: decision to have line k on or off

f_k: power flow in line k

p_g: production of generator g

θ_n: bus angle in bus n

Parameters

C_g: marginal cost of generator g

D_n: demand in bus n

P_g^+, P_g^-: minimum and maximum capacity of generator g

$F(k), T(k)$: bus that line k originates from / points to

TC_k^+, TC_k^-: max / min power flow on limit k

B_k: susceptance of line k

M_k: big-M parameter for line k

Z_k: fixed parameter indicating whether line k is on or off

References

[1] E. B. Fisher, R. P. O'Neill, and M. C. Ferris, "Optimal transmission switching," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1346–1355, August 2008.

[2] K. W. Hedman, M. C. Ferris, R. P. O'Neill, E. Bartholomew, and S. S. Oren, "Co-optimization of generation unit commitment and transmission switching with N-1 reliability," *IEEE Transactions on Power Systems*, vol. 25, no. 2, pp. 1052–1063, May 2010.

[3] J. D. Fuller, R. Ramasra, and A. Cha, "Fast heuristics for transmission- line switching," *IEEE Transactions on Power Systems*, vol. 27, no. 3, pp. 1377–1386, August 2012.

[4] D. Falcao, "High performance computing in power system applications," *Lecture Notes in Computer Science*, vol. 1215, pp. 1–23, 1997.

[5] M. V. F. Pereira, M. J. Teixeira, M. F. McCoy, and H. J. C. P. Pinto, "Developing concurrent processing applications to power system planning and operations," *IEEE Transactions on Power Systems*, vol. 5, no. 2, pp. 659–664, May 1990.

[6] A. Monticelli, M. V. F. Pereira, and S. Granville, "Security-constrained optimal power flow with post-contingency corrective rescheduling," *IEEE Transactions on Power Systems*, vol. 2, no. 1, pp. 175–180, February 1987.

[7] B. H. Kim and R. Baldick, "Coarse-grained distributed optimal power flow," *IEEE Transactions on Power Systems*, vol. 12, no. 2, pp. 932–939, May 1997.

[8] A. G. Bakirtzis and P. N. Biskas, "A decentralized solution to the DC- OPF of interconnected power systems," *IEEE Transactions on Power Systems*, vol. 18, no. 3, pp. 1007–1013, August 2003.

[9] P. N. Biskas, A. G. Bakirtzis, N. I. Macheras, and N. K. Pasialis, "A decentralized implementation of DC optimal power flow on a network of computers," *IEEE Transactions on Power Systems*, vol. 20, no. 1, pp. 25–33, February 2005.

[10] A. Papavasiliou and S. S. Oren, "Multi-area stochastic unit commitment for high wind penetration in a transmission constrained network," *Forthcoming in Operations Research*, 2011.